General Certificate of Education
January 2009
Advanced Level Examination

MATHEMATICS **MFP4**
Unit Further Pure 4

Tuesday 27 January 2009 1.30 pm to 3.00 pm

For this paper you must have:
- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The line l has equation $\mathbf{r} = (1 + 4t)\mathbf{i} + (-2 + 12t)\mathbf{j} + (1 - 3t)\mathbf{k}$.

(a) Write down a direction vector for l. *(1 mark)*

(b) (i) Find direction cosines for l. *(2 marks)*

(ii) Explain the geometrical significance of the direction cosines in relation to l. *(1 mark)*

(c) Write down a vector equation for l in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$. *(2 marks)*

2 The 2×2 matrices $\mathbf{A}$ and $\mathbf{B}$ are such that

$$\mathbf{AB} = \begin{bmatrix} 9 & 1 \\ 7 & 13 \end{bmatrix} \quad \text{and} \quad \mathbf{BA} = \begin{bmatrix} 14 & 2 \\ 1 & 8 \end{bmatrix}$$

Without finding $\mathbf{A}$ and $\mathbf{B}$:

(a) find the value of $\det \mathbf{B}$, given that $\det \mathbf{A} = 10$; *(3 marks)*

(b) determine the 2×2 matrices $\mathbf{C}$ and $\mathbf{D}$ given by

$$\mathbf{C} = (\mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}) \quad \text{and} \quad \mathbf{D} = (\mathbf{A}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}})^{\mathrm{T}}$$

where $\mathbf{M}^{\mathrm{T}}$ denotes the transpose of matrix $\mathbf{M}$. *(3 marks)*

3 The points X, Y and Z have position vectors

$$\mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathbf{z} = \begin{bmatrix} -8 \\ 1 \\ a \end{bmatrix}$$

respectively, relative to the origin O.

(a) Find:

(i) $\mathbf{x} \times \mathbf{y}$; *(2 marks)*

(ii) $(\mathbf{x} \times \mathbf{y}) \cdot \mathbf{z}$. *(2 marks)*

(b) Using these results, or otherwise, find:

(i) the area of triangle OXY; *(2 marks)*

(ii) the value of a for which $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ are linearly dependent. *(2 marks)*

4 (a) Given that -1 is an eigenvalue of the matrix $\mathbf{M} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}$, find a corresponding eigenvector. *(3 marks)*

(b) Determine the other two eigenvalues of $\mathbf{M}$, expressing each answer in its simplest surd form. *(8 marks)*

5 (a) Expand the determinant

$$D = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ z & x & y \end{vmatrix}$$

(2 marks)

(b) Show that $(x + y + z)$ is a factor of the determinant

$$\Delta = \begin{vmatrix} x & y & z \\ y - z & z - x & x - y \\ x + z & y + x & z + y \end{vmatrix}$$

(2 marks)

(c) Show that $\Delta = k(x + y + z)D$ for some integer k. *(3 marks)*

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6 The line L and the plane Π are, respectively, given by the equations

$$\mathbf{r} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathbf{r} \,.\, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 20$$

(a) Determine the size of the acute angle between L and Π. *(4 marks)*

(b) The point P has coordinates $(10, -5, 37)$.

(i) Show that P lies on L. *(1 mark)*

(ii) Find the coordinates of the point Q where L meets Π. *(4 marks)*

(iii) Deduce the distance PQ and the shortest distance from P to Π. *(3 marks)*

7 Two fixed planes have equations

$$\begin{aligned} x - 2y + z &= -1 \\ -x + y + 3z &= 3 \end{aligned}$$

(a) The point P, whose z-coordinate is λ, lies on the line of intersection of these two planes. Find the x- and y-coordinates of P in terms of λ. *(3 marks)*

(b) The point P also lies on the variable plane with equation $5x + ky + 17z = 1$. Show that

$$(k + 13)(2\lambda - 1) = 0$$ *(3 marks)*

(c) For the system of equations

$$\begin{aligned} x - 2y + z &= -1 \\ -x + y + 3z &= 3 \\ 5x + ky + 17z &= 1 \end{aligned}$$

determine the solution(s), if any, of the system, and their geometrical significance in relation to the three planes, in the cases:

(i) $k = -13$;

(ii) $k \neq -13$. *(6 marks)*

8 The plane transformation T has matrix $\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$, and maps points (x, y) onto image points (X, Y) such that

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \end{bmatrix}$$

(a) (i) Find $\mathbf{A}^{-1}$. *(2 marks)*

(ii) Hence express each of x and y in terms of X and Y. *(2 marks)*

(b) Give a full geometrical description of T. *(5 marks)*

(c) Any plane curve with equation of the form $\frac{x^2}{p} + \frac{y^2}{q} = 1$, where p and q are distinct positive constants, is an ellipse.

(i) Show that the curve E with equation $6x^2 + y^2 = 3$ is an ellipse. *(1 mark)*

(ii) Deduce that the image of the curve E under T has equation

$$2X^2 + 4XY + 5Y^2 = 15$$

(2 marks)

(iii) Explain why the curve with equation $2x^2 + 4xy + 5y^2 = 15$ is an ellipse. *(1 mark)*

END OF QUESTIONS

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